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NUMERICAL MODEL OF THE ASPIRATION SYSTEM OF AN INTERNAL COMBUSTION ENGINE

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A physicomathematical model of flow in the aspiration system of an internal combustion engine operating in a "cold" regime, i.e., for a prescribed motion of the crankshaft, has been constructed. An ideal single-species gas is used. Results of a series of calculations for a time interval of five operation cycles are presented.

1. A gas-dynamic model and a calculation algorithm for the aspiration system of a four-cycle piston internal combustion engine is proposed. The aspiration system (Fig. 1) is a combination of four channels 3 that go out from a volumetric unit 2 and shut-off valves 4 that control gas overflow to cylinders 5. A throttle valve 1 ensures contact between the volumetric unit and the power source.

The gas motion in the system is calculated using the one-dimensional channel flow equations with friction and heat exchange on the walls, and the laws of mass and energy conservation in the volumetric unit. The pressure and temperature of the gas in the cylinders and at the entrance to the system are assumed to be known. This problem was previously considered with some simplifications [1–5]. The formulation and the methods of solution of similar problems were studied [6–9].

2. A system of equations that describe a one-dimensional unsteady flow of a perfect gas in a channel with friction and heat exchange on the walls is transformed to the form

$$\frac{\partial W}{\partial t} + A\frac{\partial W}{\partial x} = f; \tag{2.1}$$

$$P = \rho RT, \tag{2.2}$$

where

$$W = |U, P, T|^{t}; \quad f = |f_1, f_2, f_3|^{t}; \qquad A = \begin{vmatrix} U & RT/P & 0 \\ \gamma P & U & 0 \\ (\gamma-1)T & 0 & U \end{vmatrix};$$

$$f_1 = -\frac{\lambda}{8}|U|U\frac{\Pi}{F}; \quad f_2 = (\gamma-1)\Big(q_0 + \frac{\lambda}{8}\rho|U|^3\Big)\frac{\Pi}{F} - \gamma\rho U\frac{\partial \ln F}{\partial x};$$

$$f_3 = (\gamma-1)\frac{T}{P}\Big(q_0 + \frac{\lambda}{8}\rho|U|^3\Big)\frac{\Pi}{F} - (\gamma-1)TU\frac{\partial \ln F}{\partial x};$$

x is the coordinate along the channel axis; F and Π are the area and the perimeter of the channel cross section; U, P, T, and ρ are the velocity, pressure, temperature, and density of the gas; q_0 is the heat flux through the side surface of the channel; λ is the skin friction coefficient; $\gamma = C_p/C_V$, $C_p = \gamma R/(\gamma-1)$, $C_V = R/(\gamma-1)$, C_p and C_V are the heat capacities of the gas at constant pressure and volume, and R is the gas constant.

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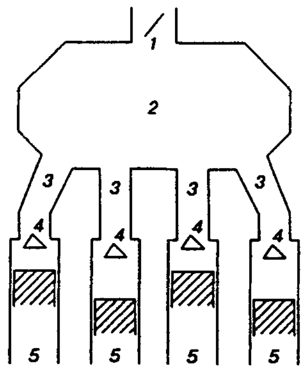

Fig. 1

To calculate the skin friction coefficient, we choose the implicit Colebrook–White approximation [10]

$$\lambda = \begin{cases} \lambda^*(\mathrm{Re}, \Delta/D) + 1/\mathrm{Re} & \text{for } \mathrm{Re} \geqslant 100, \\ \lambda^*(100, \Delta/D) + 1/\mathrm{Re} & \text{for } \mathrm{Re} < 100, \end{cases}$$

$$\lambda^* = \left[2 \log\left(\frac{2.51}{\mathrm{Re}\sqrt{\lambda^{**}}} + \frac{\Delta}{3.7D}\right)\right]^{-2}, \qquad \lambda^{**} = \left[1.8 \log\left(\frac{7}{\mathrm{Re}} + \frac{\Delta}{10D}\right)\right]^{-2}.$$

Here Δ is the height of the channel wall roughness, $D = 4F/\Pi$ is the hydraulic diameter of the channel, $\mathrm{Re} = D|U|\rho/\mu$ is the Reynolds number, and μ is the viscosity of the gas. The usual air with a temperature of 20°C was used as a test gas. In this case, we have $\mu(T) = 1.484623 \cdot 10^{-6}(T^{3/2}/(T + 117))$ kg/(m $\cdot$ sec), $R = 287.18$ m^2/(sec^2 $\cdot$ deg), and $\gamma = 1.369$ [10–12].

The system of equations (2.1) is hyperbolic, and initial and boundary conditions should be imposed. The number of boundary conditions depends on the sign of the eigenvalues of the matrix A:

$$\lambda_1 = U - C, \quad \lambda_2 = U + C, \quad \lambda_3 = U.$$

Here $C = \sqrt{\gamma RT}$ is the speed of sound. In the present paper we consider only subsonic flows, i.e., the case $|U| < C$. Thus, it is mandatory to impose one boundary condition at the left end of the channel where $\lambda_2 > 0$ and one boundary condition at the right end of the channel where $\lambda_1 < 0$. For the problem considered, it is more convenient to impose the boundary conditions for pressure. The boundary conditions for temperature are determined by the sign of the eigenvalue of $\lambda_3 = U$: if $U(0, t_n) > 0$, then $T(0, t_n)$ is assumed equal to the gas temperature in the volumetric unit; if $U(L, t_n) < 0$, where L is the channel length, then $T(L, t_n)$ is assumed equal to the gas temperature in the cylinder. Otherwise, the temperature at the channel ends is determined from additional conditions. Depending on the behavior of the solution, the total number of boundary conditions for each channel can vary from two to four. These conditions have the form of nonlinear equations.

3. The channel flow is calculated using an explicit Lax–Wendroff difference scheme [13–16] which has the second order of accuracy in internal nodes of the grid:

$$\frac{W_{i+1/2}^{n+1/2} - W_{i+1/2}^{n}}{\tau^n/2} + A(W_{i+1/2}^{n}) \frac{\partial W_{i+1/2}^{n}}{\partial x} = f(W_{i+1/2}^{n}); \tag{3.1}$$

$$\frac{W_i^{n+1} - W_i^n}{\tau^n} + A(W_i^{n+1/2}) \frac{\partial W_i^{n+1/2}}{\partial x} = f(W_i^{n+1/2}). \tag{3.2}$$

The grid is uniform in the x direction with the number of nodes N, the temporal grid t is calculated from the formula $t^{n+1} = t^n + \tau^n$, and the value of τ^n is determined from the stability condition $\tau^n x/h \leqslant \sigma < 1$ ($\sigma = \text{const}$), where $x = \max(|W_i^{n+1}| + C_i^{n+1})$. In contrast to the traditional difference scheme, the quantities $W_{i+1/2}^n$, $\partial W_{i+1/2}^n/\partial x$ in (3.1) and $W_i^{n+1/2}$, $\partial W_i^{n+1/2}/\partial x$ in (3.2) are calculated using cubic splines of the class

C^1 [17] that are constructed on grids $\{x_i\}_1^N$ and $\{x_{i+1/2} = 0.5(x_i + x_{i+1})\}_1^{N-1}$, respectively.

Approximation of the boundary conditions is performed following the pseudo-Green variant of the finite-element method [18]. The essence of this method is as follows. We consider a finite element $D1_1^n = \{x_1 \leqslant x \leqslant x_2,\ t^n \leqslant t \leqslant t^{n+1}\}$ on the left boundary of the integration domain. By transforming the coordinates $\bar{x} = (x - x_1)/(x_2 - x_1)$, $\bar{t} = (t - t^n)/\tau^n$ we transform it to a unit square $\bar{D}1_1^n = \{0 \leqslant \bar{x} \leqslant 1,\ 0 \leqslant \bar{t} \leqslant 1\}$ (the bars over the local variables $\bar{x}$ and $\bar{t}$ are omitted in what follows). According to [18], the solution of system (2.1) in the domain $\bar{D}1_1^n$ can be represented as

$$U(x,t) = U_2^{n+1}t^2 + U_1^n(1 - x) + U_2^n(x - t^2) + a_1x(1 - x) + a_2(t - t^2) + a_3(xt - t^2); \tag{3.3}$$

$$P(x,t) = P_1^{n+1}t^2 + P_1^n(1 - x - t^2) + P_2^n x + b_1x(1 - x) + b_2(t - t^2) + b_3xt. \tag{3.4}$$

The temperature $T(x,t)$ is written as $P(x,t)$, the coefficients c_j being substituted for b_j ($j = 1, 2, 3$) for $U_1^{n+1} > 0$ and as $U(x,t)$, the coefficients c_j being substituted for a_j ($j = 1, 2, 3$) for $U_1^{n+1} \leqslant 0$. The sign of U_1^{n+1} is determined in the course of solving the problem.

To find arbitrary constants a_j, b_j, and c_j ($j = 1, 2, 3$) the solution of (3.3), (3.4), and $T(x,t)$ should satisfy the system of equations (2.1) in the following collocation nodes of a boundary element:

$$(0.5, 0.21132485),\ (0.9082483, 0.78867515),\ (0.0917517, 0.78867515). \tag{3.5}$$

We obtain a system of nine linear algebraic equations. We expand the right-hand side of this system in terms of P_1^{n+1} and T_1^{n+1} and seek the solution of the system in the form of an expansion in terms of the same quantities:

$$\left.\begin{aligned} a_j &= P_1^{n+1}a_j^1 + T_1^{n+1}a_j^2 + a_j^3, \\ b_j &= P_1^{n+1}b_j^1 + T_1^{n+1}b_j^2 + b_j^3, \\ c_j &= P_1^{n+1}c_j^1 + T_1^{n+1}c_j^2 + c_j^3, \end{aligned}\right\} \quad j = 1, 2, 3. \tag{3.6}$$

As a result of the solution of three independent systems for the groups of coefficients $\{a_j^1, b_j^1, c_j^1\}$, $\{a_j^2, b_j^2, c_j^2\}$, and $\{a_j^3, b_j^3, c_j^3\}$, $j = 1, 2, 3$, we obtain the following expressions for the boundary values of velocity and temperature at the left boundary of the integration domain:

$$U_1^{n+1} = -a_3^1P_1^{n+1} - a_3^2T_1^{n+1} + (U_2^{n+1} + U_1^n - U_2^n - a_3^3)$$

for $U_1^{n+1} > 0$ (the value of T_1^{n+1} is known, it equals the gas temperature in the volumetric unit) and

$$U_1^{n+1} = -a_3^1P_1^{n+1} + U_2^{n+1} + U_1^n - U_2^n - a_3^3, \quad T_1^{n+1} = -c_3^1P_1^{n+1} + T_2^{n+1} + T_1^n - T_2^n - c_3^3$$

for $U_1^{n+1} \leqslant 0$.

The boundary conditions at the right boundary of the integration domain are determined in a similar manner. By means of a local transformation of the coordinates, the finite element $DN^n = \{x_{N-1} \leqslant x \leqslant x_N,\ t^n \leqslant t \leqslant t^{n+1}\}$ is transformed to a unit square $\bar{D}N^n = \{0 \leqslant x \leqslant 1,\ 0 \leqslant t \leqslant 1\}$, and the solution of the system of equations (2.1) is written as

$$U(x,t) = U_{N-1}^{n+1}t^2 + U_{N-1}^n(1 - x - t^2) + U_N^n x + a_1x(1 - x) + a_2(t - t^2) + a_3xt; \tag{3.7}$$

$$P(x,t) = P_N^{n+1}t^2 + P_{N-1}^n(1 - x) + P_N^n(x - t^2) + b_1x(1 - x) + b_2(t - t^2) + b_3(xt - t^2). \tag{3.8}$$

The temperature $T(x,t)$ is written as $U(x,t)$, the coefficients c_j being substituted for a_j ($j = 1, 2, 3$) for $U_N^{n+1} \geqslant 0$ and as $P(x,t)$, the coefficients c_j being substituted for b_j ($j = 1, 2, 3$) for $U_N^{n+1} < 0$. The sign of U_N^{n+1} is again determined in the course of solving the problem.

We require that functions (3.7), (3.8), and $T(x,t)$ satisfy the system of equations (2.1) in collocation nodes (3.5) of the right boundary element. As previously, we obtain a system of nine linear algebraic equations for the coefficients a_j, b_j, and c_j ($j = 1, 2, 3$). We represent these coefficients as expansions (3.6) with the quantities P_N^{n+1} and T_N^{n+1} substituted for P_1^{n+1} and T_1^{n+1}. After the corresponding systems of linear algebraic equations are solved for the coefficients a_j^l, b_j^l, c_j^l, and l ($j = 1, 2, 3$), the boundary conditions on the right boundary of the integration domain take the form

$$U_N^{n+1} = a_3^1 P_N^{n+1} + U_{N-1}^{n+1} - U_{N-1}^n + U_N^n + a_3^3, \quad T_N^{n+1} = c_3^1 P_N^{n+1} + T_{N-1}^{n+1} - T_{N-1}^n + T_N^n + c_3^3$$

for $U_N^{n+1} \geqslant 0$ and

$$U_N^{n+1} = a_3^1 P_N^{n+1} + a_3^2 T_N^{n+1} + U_{N-1}^{n+1} - U_{N-1}^n + U_N^n + a_3^3$$

for $U_N^{n+1} \leqslant 0$ (the value of T_N^{n+1} is known, it equals the gas temperature in the cylinder).

To solve the problem as a whole, we should also take into account the laws of mass and energy conservation in the volumetric unit.

4. We use the following notation: (U_0, P_0, T_0, and ρ_0) and (U_1^k, P_1^k, T_1^k, and ρ_1^k), (U_N^k, P_N^k, T_N^k, and ρ_N^k) are the velocity, pressure, temperature, and density of the gas at the entrance to the aspiration system and at the end of the kth channel ($k = 1, 2, 3, 4$); P, T, and ρ are the pressure, temperature, and density of the gas in the volumetric unit; P_k and T_k are the pressure and temperature of the gas in the kth cylinder; F_0 and F_1^k, F_N^k are the cross-section areas at the entrance to the aspiration system and at the ends of the kth channel; Π_1^k and Π_N^k are the cross-section perimeters at the ends of the kth channel; $G_0 = \rho_0 U_0 F_0$ and $G_1^k = \rho_1^k U_1^k F_1^k$, $G_N^k = \rho_N^k U_N^k F_N^k$ are the mass flow rates at the entrance to the aspiration system and at the ends of the kth channel; C_0 is the coefficient of resistance of the throttle valve; C_α, C_β^k, and C_γ^k are the coefficients of local resistance of the transitions between the throttle valve and the volumetric unit, the volumetric unit and the beginning of the kth channel, and through the valve of the kth channel.

The equations of mass and energy conservation in the volumetric unit are

$$\frac{V}{R}\frac{d}{dt}\Big(\frac{P}{T}\Big) = \rho_0 U_0 F_0 - \sum_k \rho_1^k U_1^k F_1^k; \tag{4.1}$$

$$\frac{V}{\gamma - 1}\frac{dP}{dt} = \Big(C_p T_0 + \frac{U^2}{2}\Big)\rho_0 U_0 F_0 - \sum_k \Big(C_p T_1^k + \frac{(U_1^k)^2}{2}\Big)\rho_1^k U_1^k F_1^k. \tag{4.2}$$

They should be supplemented by the conditions of pressure difference in the gas at the entrance to the system [19]

$$P_0 - P = a_0 |U_0| U_0 + b_0 U_0 \tag{4.3}$$

and the conditions of pressure difference at the transitions between the volumetric unit and the entrance to the kth channel

$$P - P_1^k = a_1^k |U_1^k| U_1^k + b_1^k U_1^k, \quad k = 1, 2, 3, 4, \tag{4.4}$$

and between the kth channel and the kth cylinder

$$P_N^k - P_k = a_N^k |U_N^k| U_N^k + b_N^k U_N^k, \quad k = 1, 2, 3, 4. \tag{4.5}$$

For the temperature and velocity at the channel ends we have

$$T_1^k = \beta_1^k T + (1 - \beta_1^k) T_1^{n+1,k}, \quad U_1^k = U_1^{n+1,k}, \quad \beta_1^k = \begin{cases} 1 & \text{if } U_1^k > 0, \\ 0 & \text{if } U_1^k \leqslant 0, \end{cases} \quad k = 1, 2, 3, 4; \tag{4.6}$$

$$\begin{gathered} T_N^k = \beta_N^k \Big(T_k - \frac{(U_N^k)^2}{2C_p}\Big) + (1 - \beta_N^k) T_N^{n+1,k}, \qquad U_N^k = U_N^{n+1,k}, \\ \beta_N^k = \begin{cases} 1 & \text{if } U_N^k > 0, \\ 0 & \text{if } U_N^k \leqslant 0, \end{cases} \quad k = 1, 2, 3, 4. \end{gathered} \tag{4.7}$$

The coefficients in these relations are determined as [19]

$$a_0 = (C_0 + C_\alpha)\rho/2, \qquad a_1^k = C_\beta^k \rho_1^k/2, \qquad a_N^k = C_\gamma^k \rho_N^k/2,$$

$$b_0 = \sqrt{\frac{\pi}{F}}\frac{\mu(T_0)}{4}, \qquad b_1^k = \frac{\Pi_1^k \mu(T_1^k)}{8F_1^k}, \qquad b_N^k = \frac{\Pi_N^k \mu(T_N^k)}{8F_N^k}.$$

TABLE 1

Range	$\lvert\Delta U/U\rvert_{max}$	$\lvert\Delta P/P\rvert_{max}$	$\lvert\Delta T/T\rvert_{max}$
$0 \leqslant x \leqslant 0.25$	$3.1 \cdot 10^{-2}$	$1.4 \cdot 10^{-4}$	$4.2 \cdot 10^{-5}$
$0.25 < x \leqslant 0.5$	$2.0 \cdot 10^{-3}$	$2.1 \cdot 10^{-4}$	$5.7 \cdot 10^{-5}$
$0.5 < x \leqslant 0.75$	$2.3 \cdot 10^{-3}$	$1.1 \cdot 10^{-4}$	$3.1 \cdot 10^{-5}$
$0.75 < x \leqslant 1$	$1.1 \cdot 10^{-3}$	$4.7 \cdot 10^{-5}$	$1.3 \cdot 10^{-5}$

The values of P_0, T_0, P_k, and T_k in relations (4.1)–(4.7) are known. Unknown quantities are U_0, P, T, U_1^k, P_1^k, T_1^k, U_N^k, P_N^k, and T_N^k ($k = 1, 2, 3, 4$). Using equalities (4.3) and (4.4) we can cancel the five unknowns U_0 and U_1^k:

$$\sigma_0 = \sqrt{b_0 + 4a_0|P_0 - P|}, \qquad U_0 = 2(P_0 - P)/(\sigma_0 + b_0),$$

$$\sigma_1^k = \sqrt{(b_1^k)^2 + 4a_1^k|P - P_1^k|}, \quad U_1^k = 2(P - P_1^k)/(\sigma_1^k + b_1^k), \qquad k = 1, 2, 3, 4.$$

Using the Newton method, we find the remaining twenty-two unknowns from Eqs. (4.1) and (4.2), equalities (4.6), and the relations

$$\frac{2(P - P_1^k)}{\sigma_1^k + b_1^k} = U_1^{n+1,k}, \qquad k = 1, 2, 3, 4.$$

Equations (4.1) and (4.2) are replaced by difference analogs. The density that enters these equations is expressed in terms of pressure and temperature in accordance with the equation of state (2.2).

The test case was chosen to be an unsteady isentropic flow in a constant-area channel in the absence of friction and heat exchange (the Riemann rarefaction wave). The calculations were performed for the following initial data: $L = 1$ m, $D = 0.05$ m, $R = 287.18$ m^2/(sec$^2 \cdot$ deg), $\gamma = 1.369$, $U_0 = 1$ m/sec, $T_0 = 1273$ K, $P_0 = 3.6558014 \cdot 10^5$ Pa, $C_0 = 707.4455$ m/sec, $t_0 = -5.13389 \cdot 10^{-3}$ sec, and $0 \leqslant t \leqslant 1.9238159 \cdot 10^{-3}$ sec.

The calculation results obtained on a uniform grid are listed in Table 1, where ΔU, ΔP, and ΔT indicate the differences between the calculated and exact values of velocity, pressure, and temperature.

5. The processes in the aspiration system of a UZAM 3417 engine were calculated using the above algorithm. The system had the following numerical characteristics: the volume of the volumetric unit $V = 5.4294 \cdot 10^{-4}$ m^3, the channel lengths $l_1 = l_4 = 0.193$ m and $l_2 = l_3 = 0.106$ m, the channel diameters $d_1 = d_2 = d_3 = d_4 = 0.034$ m, the throttle valve diameter $D_2 = 0.0405$ m, and the rotational speed $n_1 = 845$ rpm and $n_2 = 2667$ rpm. The dependence of the coefficient of resistance of the throttle valve C_α on the angle of its turning and the dependences of the coefficients of resistance of the transitions between the channel and the cylinders C_γ and the pressure in the cylinders on the crankshaft rotation angle were prescribed in a tabular form. The tabular data were obtained from experiments conducted at the Institute of Theoretical and Applied Mechanics of Siberian Division of the Russian Academy of Sciences by the team of V. K. Baev and V. V. Shumskii. The calculation was performed on a time interval of five operation cycles of the engine. A calculation with the Courant number equal to 0.7 and the number of nodes in the channels equal to 21 and 41 required about 30,000 time steps. The initial state of the gas was assumed to be an *a priori* calculated steady state that corresponds to a fully open valve of the first cylinder, the remaining valves being closed. The calculation results for the flow rate of the gas through each cylinder (valve) and the total flow rate of the gas through the system coincided with the experimental data within 7%. Particularly good agreement is observed in the middle of the operation cycle.

The study performed allows the following conclusions to be made.

• The flow in the aspiration system of an internal combustion engine is essentially unsteady and cannot be described in a quasisteady formulation.

• Different wave regimes of the flow (compression and rarefaction waves, acoustic waves, etc.) are observed in the channels of the aspiration system. These waves can interact with one another.

• The opening of aspiration valves is responsible for a significant exhaust of residual gases into the channels and the volumetric unit of the aspiration system.

• The closure of aspiration valves is responsible for the emergence of high-amplitude waves in the channels and the volumetric unit of the aspiration system. Additional studies are needed to elucidate the reasons for this phenomena.

• Despite the complex wave character of the flow, the proposed numerical model of the aspiration system allows a fairly accurate calculation of the basic characteristics of the system.

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